

Model-Independent Semileptonic Form Factors Using Dispersion Relations[§]

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1. Introduction

A non-perturbative, model-independent description of QCD form factors is a desirable ingredient for the extraction of Cabibbo-Kobayashi-Maskawa parameters from exclusive meson decays. Progress towards this goal has been realized by the development of heavy quark symmetry[1], which relates and normalizes the $\bar{B} \rightarrow D^* l \bar{\nu}$ and $\bar{B} \rightarrow D l \bar{\nu}$ form factors in the context of a $\frac{1}{M}$ expansion, where M is the heavy quark mass. Previous talks[2] have described how this normalization is used[3–5] to extract the value of the CKM parameter $|V_{cb}|$ by extrapolating the measured form factor to zero recoil, where the normalization is predicted.

This form factor extrapolation, necessary because the rate vanishes at zero recoil, introduces an uncertainty in the value of $|V_{cb}|$ due to the choice of parametrization. Estimates of this uncertainty obtained by varying parametrizations suffer the same ambiguity. This ambiguity could be eliminated if one had a non-perturbative, model-independent characterization of the form factor in terms of a small number of parameters.

In this talk we describe recent work[6–7] in which we use dispersion relations to derive such a characterization and apply it towards the extraction of $|V_{cb}|$. Our characterization of the B elastic b -number form factor uses two parameters and has 1% accuracy over the entire physical range relevant to the extraction of $|V_{cb}|$. What is important is not that we effectively find a quadratic parametrization of the Isgur-Wise function, but rather that we have determined the associated uncertainty.

The outline of the talk is as follows. We first review the expected achievable accuracy in the extraction of $|V_{cb}|$ from semi-inclusive semi-leptonic decays in Sec. 2. In Sec. 3 we review a well-known method[8] for using QCD dispersion relations and analyticity to place constraints on hadronic form factors, and show that a parametrization of the B elastic form factor $F(q^2)$ in terms of two parameters is accurate to 1% over the relevant kinematic region. In Sec. 4 we use heavy quark symmetry to relate $F(q^2)$ to the Isgur-Wise function, which describes the form factors for $\bar{B} \rightarrow D^* l \bar{\nu}$ in the infinite quark mass limit. We make a least squares fit to CLEO[3], ARGUS[4], and ALEPH[5] data using $|V_{cb}|$ and our two basis function parameters as variables, and present our results. Reliability of the method is discussed in Sec. 5, implications for $|V_{ub}|$ and $B \rightarrow K^* \gamma$ are discussed in Sec. 6, and a summary in Sec. 7.

2. Digression: $|V_{cb}|$ from inclusive semi-leptonic decays.

Several competing methods for the determination of $|V_{cb}|$ were described in previous talks[2]. We would like to discuss briefly the theoretical limitations in the determination of $|V_{cb}|$ from inclusive semi-leptonic decays. The interpretation of the measurement of the inclusive semileptonic rate $\Gamma(\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell)$ relies on our ability to calculate the rate from first principles. Using a Heavy Quark expansion one can show two things[9]:

- (i) The leading term (in $1/m_b$) is given by the parton decay rate $\Gamma(b \rightarrow c \ell \bar{\nu}_\ell)$
- (ii) There are no first order (in $1/m_b$) corrections to the previous statement.

In a $1/m_b$ expansion one may write[10,11]

$$(2.1) \quad \Gamma(\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell) = \Gamma_0 [A(x)\eta + B_K(x)K_b + B_G(x)G_b + \mathcal{O}(1/m_b^3)].$$

There is a lot of notation to explain here. First, $\Gamma_0 A(x)$ is just the parton decay rate, with $\Gamma_0 = G_F^2 m_b^5 |V_{cb}|^2 / 192 \pi^3$, $x = m_c^2 / m_b^2$ and $A(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ is a kinematic factor from the phase space integral. η is a correction factor[10] from perturbative QCD,

$$(2.2) \quad \eta = 1 - \left(\frac{\alpha}{\pi}\right) \Delta^{(1)}(x) - \left(\frac{\alpha}{\pi}\right)^2 (\beta_0 \Delta_{\beta_0}^{(2)}(x) + \dots) + \mathcal{O}(\alpha^3).$$

The one loop function $\Delta^{(1)}(x)$ can be computed analytically and equals 1.7 at $x = (0.3)^2$ (2.8 at $x = 0$). The full two-loop computation is not available, but the part proportional to β_0 , the one-loop beta function, has been computed: $\Delta_{\beta_0}^{(2)} = 1.7$ at $x = (0.3)^2$ (3.2 at $x = 0$).

The next two terms are the $1/m_b^2$ corrections, with

$$(2.3) \quad K_b = \frac{1}{2m_B} \langle B | \bar{b} \frac{D^\mu D_\mu}{2m_b^2} b | B \rangle \quad \text{and} \quad G_b = \frac{1}{2m_B} \langle B | \bar{b} \frac{g_s \sigma^{\mu\nu} G_{\mu\nu}}{4m_b^2} b | B \rangle$$

and the corresponding kinematic factors $B_K(x) = -A(x)$ and $B_G(x) = 3 - 8x + 24x^2 - 24x^3 + 5x^4 + 12x^2 \ln x$.

There are three main sources of theoretical uncertainties:

- *Quark masses* enter the rate in the combination $m_b^5 A(x)$. This is less sensitive to uncertainties in m_b than m_b^5 if m_c is fixed by

$$(2.4) \quad m_B - m_D = m_b - m_c + m_b(K_b + G_b) - m_c(K_c + G_c)$$

For a fixed guess of $K_{b,c}$ and $G_{b,c}$ Wise[10] (Shifman[11]) finds that $\Delta|V_{cb}|/|V_{cb}| = 10\%$ (3.2%) for $\Delta m_b = 0.5$ GeV (0.2 GeV).

- *Perturbative QCD corrections* are still not fully computed at two loops. This is not a limitation in principle, except in the case that the expansion does not converge quickly.¹ With $x = (0.3)^2$, $\beta_0 = 9$ and $\alpha_s = 0.2$ the three terms in (2.2) are [10] $\eta = 1 - 0.11 - 0.06$ ($\eta = 1 - 0.15 - 0.11$ for $x = 0$), which appears only marginally convergent.
- *Non-perturbative corrections* come in through the parameters K and G . The latter is well known as it determines the $B^* - B$ mass difference. Shifman[11] estimates an uncertainty of $\Delta|V_{cb}|/|V_{cb}| = 5.6\%$ from $K_b = 0.024 \pm 0.009$.

I close with a digression on a method related but not the same as the heavy quark expansion. One may argue that $\Gamma(\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell) = \Gamma(b \rightarrow c \ell \bar{\nu}_\ell)$ in the limit $m_b - m_c \ll \Lambda_{\text{QCD}} \ll m_{c,b}$. The argument is straightforward. In this limit only the D and D^* resonances are kinematically allowed, so they saturate the semi-inclusive rate. The rates into these resonances are determined effectively by the Isgur-Wise function at $v \cdot v' = 1$, where it is normalized to unity. The resulting rate is precisely $\Gamma(b \rightarrow c \ell \bar{\nu}_\ell)$, provided one does not differentiate between quark and meson masses.

In the limit, the rate for $\Gamma(\bar{B} \rightarrow X \ell \bar{\nu}_\ell) \sim (m_B - m_X)^5$, for $X = D, D^*$. This can be used to ‘explain’ the lower sensitivity of $\Gamma(\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell)$ to m_b when m_c is fixed in terms

¹ The expansion is asymptotic, so it does not converge. The asymptotic expansion has an accuracy equal to the size of the term at which the expansion is truncated.

of m_b using the known value of $m_B - m_D$. But the real question is whether this could be the basis for a systematic expansion. Note, however, that

$$(m_B - m_D)^5 = (m_b - m_c)^5(1 + K + G)^5 + \dots \quad (2.5a)$$

$$(m_B - m_{D^*})^5 = (m_b - m_c)^5(1 + K - G)^5 \left(1 + 10G \frac{m_b + m_c}{m_b - m_c}\right) + \dots \quad (2.5b)$$

These correction factors will enter the relation between inclusive and partonic widths. The expansion parameter in Eq. (2.5b) is poor: $G \frac{m_b + m_c}{m_b - m_c} \sim \frac{\Lambda_{\text{QCD}}}{m_b - m_c} \frac{\Lambda_{\text{QCD}}}{m_b}$.

3. The Analyticity Constraints²

Of primary interest is the form factor F , defined by

$$(3.1) \quad \langle B(p') | V_\mu | B(p) \rangle = F(q^2)(p + p')_\mu,$$

where $V_\mu = \bar{b}\gamma_\mu b$, and $q^2 = (p - p')^2$. Crossing symmetry states that the form factor F is a function of a complex variable, q^2 , which gives the matrix element in Eq. (3.1) for real negative q^2 , and the matrix element $\langle 0 | V_\mu | B\bar{B} \rangle$ for real $q^2 \geq 4m_B^2$.

Consider the two-point function

$$(3.2) \quad i \int d^4x e^{iqx} \langle T V_\mu(x) V_\nu^\dagger(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

In QCD the structure functions satisfy a once-subtracted dispersion relation:

$$(3.3) \quad \frac{\partial \Pi}{\partial q^2} \Big|_{q^2 = -Q^2} = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im } \Pi(t)}{(t + Q^2)^2}.$$

The absorptive part $\text{Im } \Pi(q^2)$ is obtained by inserting real states between the two currents on the right-hand side of Eq. (3.2). This is a sum of positive definite terms, so one can obtain strict inequalities by concentrating on the term with intermediate states of $B\bar{B}$ pairs. For Q^2 far from the resonance region ($Q^2 + 4m_b^2 \gg m_b \Lambda_{\text{QCD}}$) the two-point function can be computed reliably from perturbative QCD. We set $Q^2 = 0$ which for large b quark mass is far from resonances. One thus obtains an inequality of the form

$$(3.4) \quad \int_{4M_B^2}^\infty dt k(t) |F|^2 \leq 1,$$

² This section based on refs. [8,12–14]

where the function $k(t)$ is the ratio of the kinematic factor on the right hand side of Eq. (3.3) to the QCD calculation of the left hand side.

A key ingredient in this approach is the transformation that maps the complex q^2 plane onto the unit disc $|z| \leq 1$:

$$(3.5) \quad \sqrt{1 - \frac{q^2}{4M_B^2}} = \frac{1+z}{1-z} .$$

In terms of the angular variable $e^{i\theta} \equiv z$, the once-subtracted QCD dispersion relation may be written as

$$(3.6) \quad \frac{1}{2\pi} \int_0^{2\pi} d\theta |\phi(e^{i\theta})F(e^{i\theta})|^2 \leq \frac{1}{\pi} ,$$

where the weighing function $\phi(e^{i\theta}) = k(t(\theta))dt/d\theta$ [12]:

$$(3.7) \quad \phi(z) = \frac{1}{16} \sqrt{\frac{5n_f}{6\rho}} (1+z)^2 \sqrt{1-z} .$$

Here n_f is the number of light flavors for which $SU(n_f)$ flavor symmetry is valid; we take $n_f = 2$. Perturbative corrections to the dispersion relation are incorporated in ρ , which has been computed[15] to $\mathcal{O}(\alpha_s)$, $\rho = 1 + 0.73\alpha_s(m_b) \approx 1.20$.

Note that $\phi(z)$ is analytic in $|z| < 1$, while poles of F inside the unit disc originate from resonances below threshold and cannot be ignored[13]. In this talk we only consider the effects of the resonances $\Upsilon_{1,2,3}$. Although cuts below threshold should be considered, they are expected to have smaller physical effects. A simple but effective trick[14] eliminates the poles with no reference to the size of their residues but rather only their positions (*i.e.*, masses). The function

$$(3.8) \quad P(z) \equiv \frac{(z-z_1)(z-z_2)(z-z_3)}{(1-\bar{z}_1z)(1-\bar{z}_2z)(1-\bar{z}_3z)} ,$$

where the z_i correspond to the values $q^2 = M_{\Upsilon_i}^2$, is analytic in $|z| \leq 1$ and satisfies $|P(z)| = 1$ for $|z| = 1$. The function $P(z)\phi(z)F(z)$ is analytic on the unit disk, and obeys

$$(3.9) \quad \frac{1}{2\pi} \int_0^{2\pi} d\theta |P(e^{i\theta})\phi(e^{i\theta})F(e^{i\theta})|^2 \leq \frac{1}{\pi} .$$

It follows that the QCD form factor may therefore be written as

$$(3.10a, b) \quad F(z) = \frac{1}{P(z)\phi(z)} \sum_{n=0}^{\infty} a_n z^n \quad \text{with} \quad \sum_{n=0}^{\infty} |a_n|^2 \leq \frac{1}{\pi}$$

Since B -number is conserved, $F(0) = 1$, so that $a_0 = P(0)\phi(0)$.

In the next section we will use heavy quark symmetries to relate F to the form factors for $\bar{B} \rightarrow D^* l \bar{\nu}$, where the physical kinematic range is $0 < z < 0.056$. Therefore, the form factor in (3.10a) converges quickly over the physical region. Note that for this it is crucial that the coefficients a_n be bounded as in (3.10b). Retaining only a_1 and a_2 in Eq. (3.10a), we have

$$(3.11) \quad F(z) = \frac{1}{P(z)\phi(z)} [P(0)\phi(0) + a_1 z + a_2 z^2]$$

with a maximum relative error of $\sim \sqrt{1/\pi}(0.056)^3/P(0)\phi(0) \approx 0.01$.

4. Extraction of $|V_{cb}|$

4.1. Heavy Quark Symmetry Relations

In the infinite b and c quark mass limit all the form factors for $\bar{B} \rightarrow D l \bar{\nu}$ and $\bar{B} \rightarrow D^* l \bar{\nu}$ are given by one universal ‘‘Isgur-Wise’’ function. This allows us to apply the constraint on F to the particular combination of form factors actually measured, rather than deriving constraints for each form factor separately. For large b -mass, the Isgur-Wise function is given by the form factor F , $F(\omega) = \xi(\omega)$, where $\omega = v \cdot v' = p_B \cdot p_D / m_B m_D$. There are short-distance matching and running corrections[16] to the relation between $\xi(\omega)$ and the $\bar{B} \rightarrow D^*$ form factors. For example, for the vector current form factor g one may write $g(z) = \eta_D F(z)$. This relation generally holds to order $1/M$, but at threshold it holds to order $1/M^2$ [17]. We take $\eta_D = 0.985$.

4.2. Maximum Likelihood Fit

Once the essential physics of QCD is incorporated into the calculation via Eq. (3.11), the maximum likelihood fit is simply an ordinary chi-squared minimization with parameters $|V_{cb}|$, a_1 and a_2 . We normalize input data to a B lifetime[18] of $\tau_B = 1.61$ ps. Also, we rescale the ARGUS data to reflect a revised branching fraction for $D^0 \rightarrow K^- \pi^+$.

Table 1 shows the central values and 68% confidence levels for $|V_{cb}|$, a_1 , and a_2 from the various experiments. That a_2 saturates the bound $|a_2| \leq 0.55$ is not significant because its variance is large. Figure 1 shows the product of the best fit form factors with $|V_{cb}|$, superimposed with experimental data. At 90% confidence level, a_1 and a_2 are consistent with zero, suggesting the dispersion relation may be saturated entirely by higher states.

$ V_{cb} \cdot 10^3$	a_1	a_2	Expt.
$35.7^{+4.2}_{-2.8}$	$0.00^{+0.02}_{-0.07}$	$-0.55^{+1.1}_{-0.0}$	CLEO[3]
$45.4^{+7.5}_{-10.7}$	$-0.11^{+0.10}_{-0.03}$	$0.55^{+0.0}_{-1.1}$	ARGUS[4]
$32.2^{+4.58}_{-5.89}$	$0.00^{+0.11}_{-0.04}$	$0.45^{+0.1}_{-1.1}$	ALEPH[5]

Table 1. *Fit values for $|V_{cb}|$, a_1 , and a_2 from the various experiments.*

The errors on $|V_{cb}|$ in Table 1 are statistical only; the treatment of systematic errors depends both on our parametrization and a detailed understanding of the experiment. The error implicit in the variation over choices of parametrization, however, is absent. ARGUS examined the effect of varying over four possible parametrizations, which induced a spread of 0.012 in $|V_{cb}|$, and was the major impediment in using heavy quark symmetry to obtain a model-independent extraction.

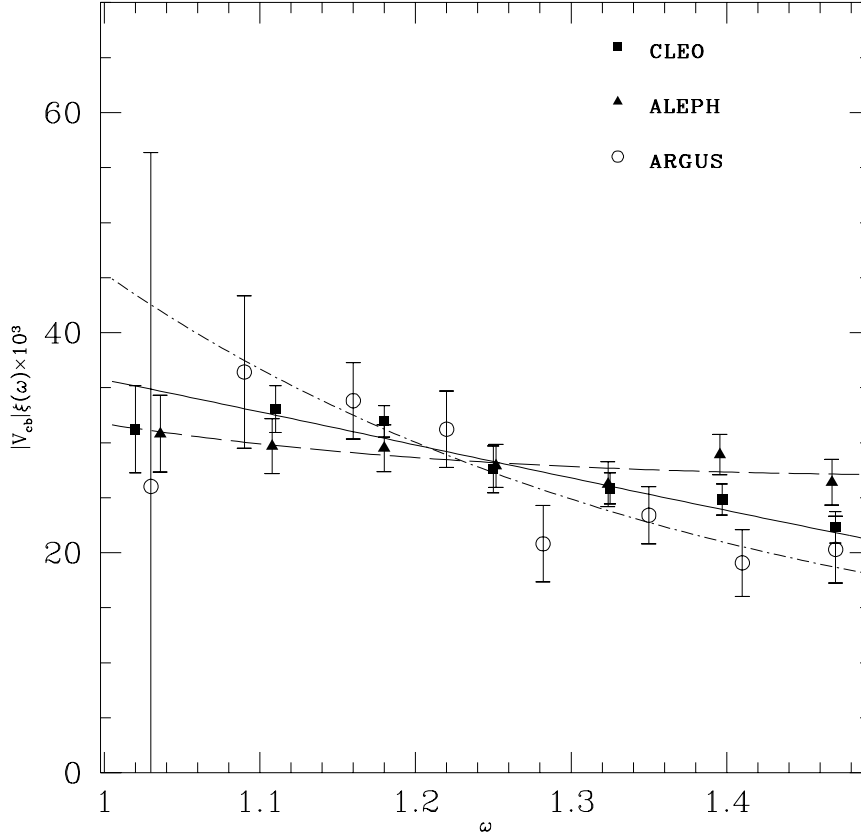


Figure 1. *Best fit values for CLEO (solid line), ARGUS (dashed line), and ALEPH (dash-dotted line). The data for each experiment is shown with error bars. The parametrization used in the text, is shown with a solid line to reflect a revised branching ratio.*

5. Discussion of the Method

Perturbative and non-perturbative corrections to the dispersion relation (3.9) enter our analysis by modifying the value of the parameter ρ in Eq. (3.7). The extraction of

$|V_{cb}|$ is rather insensitive to such corrections. A change of ρ by $\pm 10\%$ changes the central value of $|V_{cb}|$ by less than $\pm 0.3\%$.

The truncation of Eq. (3.10a) to the first N terms introduces an error proportional to $\frac{1}{P(0)\phi(0)} \sum_{n=N+1}^{\infty} a_n z^n < 1\%$, a bound valid over the whole physical region $0 \leq z \leq 0.056$.

The application of the $B \rightarrow B$ dispersion relation to $\bar{B} \rightarrow D^* l \bar{\nu}$ decays relies on heavy quark symmetry. This is potentially the largest source of error, of $\mathcal{O}(\frac{1}{M})$. We estimate such corrections by making a 20% change in the ranges of a_1 and a_2 , resulting in a 2% shift in the central value of $|V_{cb}|$. One can derive analogous dispersion relations for each of the $\bar{B} \rightarrow D^* l \bar{\nu}$ form factors, sidestepping the need for heavy quark symmetry (except at q_{max}^2)[19].

Another heavy quark correction arises at $\mathcal{O}(\frac{1}{M^2})$ in the normalization of the Isgur-Wise function at threshold. The normalization of the form factor $g(\omega = 1)$ has been estimated to be $g(1) = 0.96$ [20], $g(1) = 0.89$ [21], and $g(1) = 0.93$ [22]. We have included a QCD correction of 0.985, so to good approximation, this simply rescales the values of $|V_{cb}|$ in Table 1 by $\frac{0.985}{g(1)}$.

There are other errors in our extraction that are not purely theoretical. The most pressing of these involve the binning of the measured rate against ω , smearing of ω introduced by boosting from the lab to the center of mass frame, and correlation of errors. Randomly varying input values of ω in our least squares fit of the CLEO data by ± 0.05 changes the central value of $|V_{cb}|$ by less than 1%. A more thorough extraction can be done by the experimental groups themselves, using our parametrization in their analysis.

6. Other Applications.

Our parametrization of form factors applies to other heavy hadron decays, including $\bar{B} \rightarrow \pi l \bar{\nu}$, with minor modifications. In this case the range of the kinematic variable is larger, $0 < z < 0.5$, so more coefficients a_n are needed for comparable accuracy. We expect six to eight a_n will be necessary for accuracy of a few percent over the entire kinematic range, depending on the form of the actual data.

Heavy quark symmetry relates the form factors for $B \rightarrow \pi l \bar{\nu}$ and $D \rightarrow \pi l \bar{\nu}$. The latter is readily measured over $0 < z < 0.3$, and our method then allows a reliable extrapolation to $0.3 \leq z < 0.5$. This opens up the possibility of reliably extracting $|V_{ub}|$ from a measurement of $\text{Br}(B \rightarrow \pi l \bar{\nu})$.

An extraction of a model-independent lower bound on $|V_{ub}|$ should be possible since small values of $|V_{ub}|$ tend to wash out the nontrivial z dependence, while the a_n 's cannot compensate because they are bounded from above.

Any model of hadronic form factors must predict a form factor that is consistent with our parametrization. This is a severe test to pass, and serves as an effective discriminator for models[7].

Our parametrization may also be useful in the analysis of $B \rightarrow K^*\gamma$, by relating its amplitude to the form factor for $D \rightarrow K^*e\nu$ extrapolated outside the physical region[23].

7. Summary

The extraction of the CKM mixing parameter $|V_{cb}|$ involves several types of uncertainties. Typically, these uncertainties are classified as

$$(7.1) \quad |V_{cb}| = V \pm \{stat\} \pm \{syst\} \pm \{life\} \pm \{norm\} \pm \{param\}$$

where *stat* and *syst* refer to statistical and systematic experimental uncertainties, *life* refers to uncertainties in the B lifetime, *norm* refers to uncertainty in the value of the form factor at threshold, and *param* refers to uncertainty in the extrapolation of the measured differential rate to threshold.

Not only the central value, but also the statistical uncertainty depends on the parametrization. For example, linear fits to CLEO data yield substantially smaller statistical uncertainties than quadratic fits. Typically, quoted values correspond to the parametrization yielding the smallest statistical uncertainty, in effect throwing some statistical uncertainty into the parametrization uncertainty, which remains implicit. Clearly, this does not improve the accuracy with which we know $|V_{cb}|$.

We have essentially eliminated the uncertainty in the choice of parametrization. This was accomplished in four stages. First, we used QCD dispersion relations to constrain the $B \rightarrow B$ elastic form factor. Second, we derived a general parametrization of the B elastic form factor that automatically satisfies the dispersion relation constraint. This expression involved an infinite number of parameters a_n bounded by $\sum_{n=0}^{\infty} |a_n|^2 \leq I$. Third, we used heavy quark symmetry to relate the B elastic form factor to $\bar{B} \rightarrow D^*l\bar{\nu}$ form factors and fixed the normalization at threshold. Over the entire kinematic range relevant to $\bar{B} \rightarrow D^*l\bar{\nu}$, we showed that neglecting all but the first two parameters a_1, a_2 resulted in

at most a 1% deviation in the predicted form factor. Finally, we made a least squares fit of the differential $\bar{B} \rightarrow D^* l \bar{\nu}$ rate to $|V_{cb}|$, a_1 , and a_2 .

The results of this fit improve on all previous extractions in one important way: The uncertainty due to the choice of parametrization is under control. Our statistical errors are larger than many quoted values. This does not reflect an inferiority of our method, but rather quantifies uncertainties that were previously left implicit. Our averaged value from CLEO, ARGUS, and ALEPH data is

$$(7.2) \qquad |V_{cb}| \times 10^3 = 35.5^{+2.9}_{-2.5} (stat).$$

An estimation of systematic uncertainties requires a detailed knowledge of the experiments.

Our parametrization may be useful for other processes, such as $\bar{B} \rightarrow \pi l \bar{\nu}$ and $B \rightarrow K^* \gamma$ as well. For $\bar{B} \rightarrow \pi l \bar{\nu}$ we expect to be able to extract a model independent lower bound on $|V_{ub}|$, and a good measure of $|V_{ub}|$ by use of heavy quark symmetries to infer the form factor from $D \rightarrow \pi l \bar{\nu}$ by extrapolation. In addition, precision tests of QCD-predicted form factors are now possible; these should be useful as checks of QCD models and lattice simulations.

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